Unit 13: Bernoulli, Binomial, Geometric and Poisson Distributions and their Applications

13.1 Random variable, probability function, and discrete probability distribution

(i) The concept of random variable should be introduced through daily life examples.

(ii) Discrete random variable is to be discussed. Continuous one is left until normal distribution is introduced.

Fundamental difference between the discrete and continuous random variables :

discrete - data related to numerical responses that arise from a counting process, for example,
(a) total scores in a basketball match
(b) number of road accidents in a city within an hour
(c) number of magazines subscribed yearly

continuous - data related to numerical responses that arise from a measuring process, for example,
(a) hours of sunset
(b) average weights of packets of rice
(c) body temperature

(iii) Discrete probability distribution can be illustrated with simple examples such as:
(a) number of misprints on a page of a book
(b) number of left-handed students in each class.

(iv) The definition and concept of the mean and variance of a random variable should be introduced. These two measures should be related to measures of central tendency and dispersion to reinforce students' understanding.

13.2 Bernoulli distribution

Bernoulli trials refer to experiments having two outcomes only, usually called “success” and “failure”. Daily life examples should be given.

(i) A student without studying guesses the answer for a multiple choice question.
(ii) A randomly chosen student in the class is left-handed.

Mathematically, the probability function of Bernoulli distribution is given by

$$f(x;\ p) = p^x(1-p)^{1-x} \quad \text{for } x = 0, 1$$

where p is the probability of success.

mean = p ; variance = $p(1-p)$.

13.3 Binomial distribution

Binomial distribution will arise from n independent and identical Bernoulli trials with the same probability of success (p) for each trial. For example,

The distribution of the number of correct answers guessed by a student for a multiple choice examination of 10 questions.

Mathematically, the probability function of binomial distribution is given by

$$f(x;\ n,\ p) = C_x^n p^x (1-p)^{n-x} \quad \text{for } x = 0, 1, 2, \ldots, n.$$

mean = np, variance = $\sigma^2 = np(1-p)$. Proof is not expected.
The notation Bi (n, p) or Bi (x; n, p) may be introduced.

Use of binomial distribution table is not expected.

13.4 Geometric distribution

The idea of geometric distribution is to get the first success in the xth trial of independent and identical Bernoulli trials. For example,

If the xth question is the first question correctly guessed by a student in a multiple choice paper, the distribution of x is a geometric distribution.

Mathematically, the probability function of geometric distribution is given by

$$f(x;\ p) = p(1-p)^{x-1} \quad \text{for } x = 1, 2, \ldots$$

mean $= \mu = \dfrac{1}{p}$, variance $= \sigma^2 = \dfrac{1-p}{p^2}$. Proof is not expected.

Students should note the difference between geometric distribution and binomial distribution:

Carry out x trials until first success occurs $\rightarrow$ geometric distribution
Number of x successes in n trials $\rightarrow$ binomial distribution

13.5 Poisson distribution

Poisson distribution mainly concerns about the probability distribution of the number of events occurring within a time interval or space. Daily life examples are :

(i) number of cars arriving at a petrol station in anyone minute
(ii) number of passengers joining a queue for a bus in any five minutes during rush hours
(iii) number of telephone calls coming into a switchboard in any 30 seconds period
(iv) number of misprints per page of a book
(v) number of flying bombs hitting the target

Mathematically, the probability function of Poisson distribution is given by

$$f(x;\ \lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad \text{where } x = 0, 1, 2, \ldots; \quad \lambda > 0.$$

The notation Po(x ; λ) or Po(λ) may be used.

mean = λ, variance = $\sigma^2 = \lambda$. Proof is not expected.

In general, Poisson distribution is based on the following assumptions:

(i) events occur randomly over time or space;
(ii) at any instant, there is either one occurrence or no occurrence of the event;.
(iii) events occur at a constant rate, λ, over time or space;
(iv) events occur independently.

Poisson distribution is actually a binomial distribution under the limiting condition that $n \to \infty$ and $p \to 0$ with $np = \lambda$ = constant.

Use of Poisson table is not expected.

13.6 Applications of Bernoulli, Binomial, Geometric and Poisson distributions

Throughout this unit, daily life examples and discussions are the essential features.

Binomial Distribution

Example 1
The probability that a salesperson will sell a magazine subscription to someone who has been randomly selected from the telephone directory is 0.1. If the salesperson calls 6 individuals this evening, what is the probability that
(i) there will be no subscription sold?
(ii) exactly 3 subscriptions will be sold?
(iii) at least 3 subscriptions will be sold?
(iv) at most 3 subscriptions will be sold?

Geometric Distribution

Example 1
The probability that a student will pass a test on any trial is 0.6. What is the probability that he will eventually pass the test on the second trial?

Example 2
Suppose the probability that Hong Kong Observatory will make correct daily weather forecasts is 0.8. In the coming days, what is the probability that it will make the first correct forecast on the fourth day?

Poisson Distribution

Example 1

The average number of claims per day made to the Insurance Company for damage or losses is 3.1. What is the probability that in any given day

(i) fewer than 2 claims will be made?
(ii) exactly 2 claims will be made?
(iii) 2 or more claims will be made?
(iv) more than 2 claims will be made?

Example 2

Based on past experience, 1 % of the telephone bills mailed to house-holds in Hong Kong are incorrect. If a sample of 10 bills is selected, find the probability that at least one bill will be incorrect. Do this using two probability distributions (the binomial and the Poisson) and briefly compare and explain your results.