Unit 5: Limit and derivative

5.1 Limit of a function

In this section, the concepts and notations of functions and limits of functions are to be taught only with an intuitive approach.

Teacher may use examples such as the energy level of recharging a mobile phone battery, the speed limit of an accelerating particle and bacteria growth etc. to introduce the concept of limit. Tables and graphs showing infinitesimal changes may help to illustrate the meaning of the limit of $f(x)$ as x approaches the value a, which is denoted by $\lim_{x \to a} f(x) = L$ The ε - δ approach should be avoided. Use of computer would be a convincing demonstration. Students should understand that the function exists no limit at a if $f(x) \to \pm\infty$ as x tends to a.

Examples on the existence of a limit and the non-existence of a limit such as $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$ and $\lim_{x \to 5} \frac{x^2 - 25}{|x - 5|}$ should be discussed with students. Students may note that although $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$ exists, but the function $f(x) = \frac{x^2 - 25}{x - 5}$ is not defined at x = 5. Limits of function $f(x)$ as x tends to ∞ should also be mentioned.

Limits on trigonometric functions are not required.

Theorems on limits of sum, difference, product, quotient, scalar multiplication and composite functions should be stated without proof Students should only be expected to grasp idea of the limit of a function in order to understand the concept of derivative of a function at later stage.

The limit of $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ as $h \to 0$ could be discussed to facilitate a smooth introduction to the concept of differentiation.

<u>Example 1</u>
The energy level (in Joules) of recharging a battery is assumed to be described by the function $E(t) = 10\left(1 - e^{-\frac{t}{2}}\right)$, where t denotes the number of hours of recharging the battery. Determine the energy levels of the battery after recharging 1 hour, 2 hours and 4 hours. What is the eventual energy level?

5.2 Derivative of a function

Common notations of derivatives such as $f'(x)$, $\frac{\mathrm{d}y}{\mathrm{d}x}$, y', $\frac{\mathrm{d}f(x)}{\mathrm{d}x}$ and $\frac{\mathrm{d}}{\mathrm{d}x}f(x)$ should be introduced. Teachers should emphasize that $\frac{\mathrm{d}y}{\mathrm{d}x}$ is a symbol and should not be taken as a fraction.

Teachers are expected to demonstrate the methods in finding derivatives from first principles of the following functions, such as x^4; $\sqrt{x+1}$; $x^2(x+2)$ and $\frac{1}{x-1}$.

Students should know how to find the derivatives of simple functions. Moreover, students are expected to know the difference between $f'(x)$ and $f'(a)$, where a is a known value.